OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4756**

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** Given that $y = \arctan\sqrt{x}$, find $\frac{dy}{dx}$, giving your answer in terms of x. Hence show that

$$\int_0^1 \frac{1}{\sqrt{x}(x+1)}\,dx = \frac{\pi}{2}.$$ **[6]**

(b) A curve has cartesian equation

$$x^2 + y^2 = xy + 1.$$

(i) Show that the polar equation of the curve is

$$r^2 = \frac{2}{2 - \sin 2\theta}.$$ **[4]**

(ii) Determine the greatest and least positive values of r and the values of θ between 0 and 2π for which they occur. **[6]**

(iii) Sketch the curve. **[2]**

2 **(a)** Use de Moivre's theorem to find the constants a, b, c in the identity

$$\cos 5\theta \equiv a\cos^5\theta + b\cos^3\theta + c\cos\theta.$$ **[6]**

(b) Let

$$C = \cos\theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \ldots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

$$\text{and } S = \sin\theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \ldots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

where n is an integer greater than 1.

By considering $C + \mathrm{j}S$, show that $C = 0$ and $S = 0$. **[7]**

(c) Write down the Maclaurin series for e^t as far as the term in t^2.

Hence show that, for t close to zero,

$$\frac{t}{e^t - 1} \approx 1 - \tfrac{1}{2}t.$$ **[5]**

3 **(i)** Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & a \\ 2 & -1 & 2 \\ 3 & -2 & 2 \end{pmatrix}$$

where $a \neq 4$.

Show that when $a = -1$ the inverse is

$$\frac{1}{5}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 5 & -4 \\ -1 & 5 & -3 \end{pmatrix}.$$ **[6]**

(ii) Solve, in terms of b, the following system of equations. **[5]**

$$\begin{aligned} x + y - z &= -2 \\ 2x - y + 2z &= b \\ 3x - 2y + 2z &= 1 \end{aligned}$$

(iii) Find the value of b for which the equations

$$\begin{aligned} x + y + 4z &= -2 \\ 2x - y + 2z &= b \\ 3x - 2y + 2z &= 1 \end{aligned}$$

have solutions. Give a geometrical interpretation of the solutions in this case. **[7]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Prove, using exponential functions, that

$$\cosh 2x = 1 + 2\sinh^2 x.$$

Differentiate this result to obtain a formula for $\sinh 2x$. **[4]**

(ii) Solve the equation

$$2\cosh 2x + 3\sinh x = 3,$$

expressing your answers in exact logarithmic form. **[7]**

(iii) Given that $\cosh t = \frac{5}{4}$, show by using exponential functions that $t = \pm \ln 2$.

Find the exact value of the integral

$$\int_4^5 \frac{1}{\sqrt{x^2 - 16}}\,dx.$$ **[7]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A line PQ is of length k (where $k > 1$) and it passes through the point $(1, 0)$. PQ is inclined at angle θ to the positive x-axis. The end Q moves along the y-axis. See Fig. 5. The end P traces out a locus.

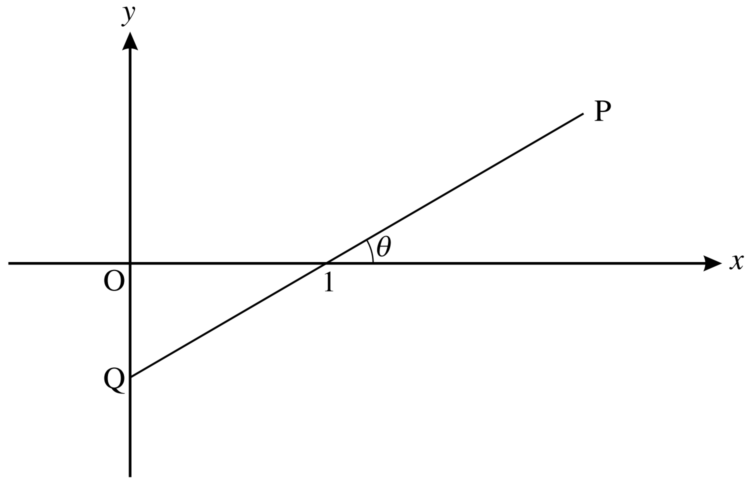

Fig. 5

(i) Show that the locus of P may be expressed parametrically as follows. **[3]**

$$x = k\cos\theta \qquad y = k\sin\theta - \tan\theta$$

You are now required to investigate curves with these parametric equations, where k may take any non-zero value and $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(ii) Use your calculator to sketch the curve in each of the cases $k = 2$, $k = 1$, $k = \frac{1}{2}$ and $k = -1$. **[4]**

(iii) For what value(s) of k does the curve have

(A) an asymptote (you should state what the asymptote is),

(B) a cusp,

(C) a loop? **[3]**

(iv) For the case $k = 2$, find the angle at which the curve crosses itself. **[2]**

(v) For the case $k = 8$, find in an exact form the coordinates of the highest point on the loop. **[3]**

(vi) Verify that the cartesian equation of the curve is

$$y^2 = \frac{(x-1)^2}{x^2}(k^2 - x^2).$$ **[3]**

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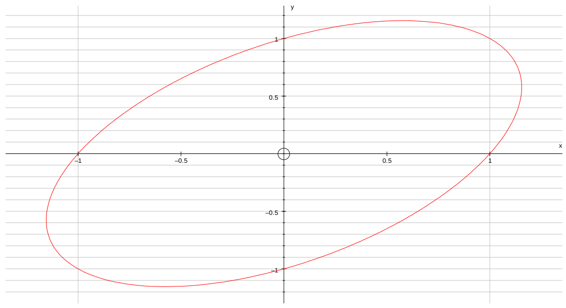
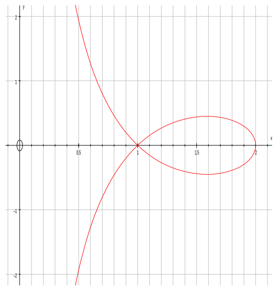
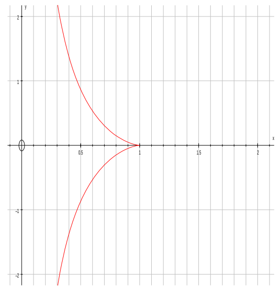
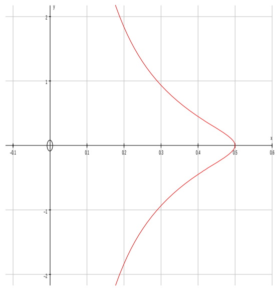
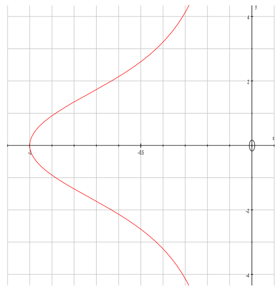
1 (a)	$y = \arctan\sqrt{x}$ $u = \sqrt{x}$, $y = \arctan u$ $\Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$, $\frac{dy}{du} = \frac{1}{1+u^2}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{1+u^2} \times \frac{1}{2\sqrt{x}}$ $= \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(x+1)}$	M1 A1 A1	Using Chain Rule Correct derivative in any form Correct derivative in terms of x
	OR $\tan y = \sqrt{x}$ $\Rightarrow \sec^2 y \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ M1A1 $\sec^2 y = 1 + \tan^2 y = 1 + x$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}(x+1)}$ A1		Rearranging for $\sqrt{x}$ or x and differentiating implicitly
	$\Rightarrow \int_0^1 \frac{1}{\sqrt{x}(x+1)} dx = \left[2\arctan\sqrt{x}\right]_0^1$ $= 2 \arctan 1 - 2 \arctan 0$ $= 2 \times \frac{\pi}{4} = \frac{\pi}{2}$	M1 A1 A1 (ag)	Integral in form $k \arctan\sqrt{x}$ $k = 2$
(b)(i)	$x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$ $x^2 + y^2 = xy + 1$ $\Rightarrow r^2 = r^2\cos\theta\sin\theta + 1$ $\Rightarrow r^2 = \frac{1}{2}r^2\sin 2\theta + 1$ $\Rightarrow 2r^2 = r^2\sin 2\theta + 2$ $\Rightarrow r^2(2 - \sin 2\theta) = 2$ $\Rightarrow r^2 = \frac{2}{2 - \sin 2\theta}$	M1 A1 A1 A1 (ag) **4**	Using at least one of these LHS RHS Clearly obtained *SR*: $x = r\sin\theta$, $y = r\cos\theta$ used M1A1A0A0 max.
(ii)	Max r is $\sqrt{2}$ Occurs when $\sin 2\theta = 1$ $\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$ Min $r = \sqrt{\frac{2}{3}}$ Occurs when $\sin 2\theta = -1$ $\Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$	B1 M1 A1 B1 M1 A1 **6**	 Attempting to solve Both. Accept degrees. A0 if extras in range $\frac{\sqrt{6}}{3}$ Attempting to solve (must be −1) Both. Accept degrees. A0 if extras in range

(iii)		G1 G1 **2**	Closed curve, roughly elliptical, with no points or dents Major axis along $y = x$ **18**
2 (a)	$\cos 5\theta + \mathrm{j}\sin 5\theta = (\cos\theta + \mathrm{j}\sin\theta)^5$ $= \cos^5\theta + 5\cos^4\theta\,\mathrm{j}\sin\theta + 10\cos^3\theta\,\mathrm{j}^2\sin^2\theta$ $+ 10\cos^2\theta\,\mathrm{j}^3\sin^3\theta + 5\cos\theta\,\mathrm{j}^4\sin^4\theta + \mathrm{j}^5\sin^5\theta$ $= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta + \mathrm{j}(\ldots)$ $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	M1 M1 A1 M1 M1 A1 **6**	Using de Moivre Using binomial theorem appropriately Correct real part. Must evaluate powers of j Equating real parts Replacing $\sin^2\theta$ by $1 - \cos^2\theta$ $a = 16$, $b = -20$, $c = 5$
(b)	$C + \mathrm{j}S$ $= e^{\mathrm{j}\theta} + e^{\mathrm{j}\left(\theta + \frac{2\pi}{n}\right)} + \ldots + e^{\mathrm{j}\left(\theta + \frac{(2n-2)\pi}{n}\right)}$ This is a G.P. $a = e^{\mathrm{j}\theta}$, $r = e^{\mathrm{j}\frac{2\pi}{n}}$ Sum $= \dfrac{e^{\mathrm{j}\theta}\left(1 - \left(e^{\mathrm{j}\frac{2\pi}{n}}\right)^n\right)}{1 - e^{\mathrm{j}\frac{2\pi}{n}}}$ Numerator $= e^{\mathrm{j}\theta}\left(1 - e^{2\pi \mathrm{j}}\right)$ and $e^{2\pi \mathrm{j}} = 1$ so sum = 0 $\Rightarrow\quad C = 0$ and $S = 0$	M1 A1 M1 A1 A1 E1 E1 **7**	Forming series $C + \mathrm{j}S$ as exponentials Need not see whole series Attempting to sum finite or infinite G.P. Correct a, r used or stated, and n terms Must see j Convincing explanation that sum = 0 $C = S = 0$. Dep. on previous E1 Both E marks dep. on 5 marks above
(c)	$e^t \approx 1 + t + \frac{1}{2}t^2$ $\dfrac{t}{e^t - 1} \approx \dfrac{t}{t + \frac{1}{2}t^2}$ $\dfrac{t}{t + \frac{1}{2}t^2} = \dfrac{1}{1 + \frac{1}{2}t} = \left(1 + \frac{1}{2}t\right)^{-1} = 1 - \frac{1}{2}t + \ldots$	B1 M1 A1 M1	Ignore terms in higher powers Substituting Maclaurin series Suitable manipulation and use of binomial theorem
	OR $\dfrac{1}{1 + \frac{1}{2}t} = \dfrac{1}{1 + \frac{1}{2}t} \times \dfrac{1 - \frac{1}{2}t}{1 - \frac{1}{2}t} = \dfrac{1 - \frac{1}{2}t}{1 - \frac{1}{4}t^2}$ M1		
	Hence $\dfrac{t}{e^t - 1} \approx 1 - \frac{1}{2}t$	A1 (ag)	
	OR $\left(e^t - 1\right)\left(1 - \frac{1}{2}t\right) = \left(t + \frac{1}{2}t^2 + \ldots\right)\left(1 - \frac{1}{2}t\right)$ M1 A1 $\approx t +$ terms in t^3 M1 $\Rightarrow\quad \dfrac{t}{e^t - 1} \approx 1 - \frac{1}{2}t$ A1		Substituting Maclaurin series Correct expression Multiplying out Convincing explanation
		5	**18**

3 (i)	$\mathbf{M}^{-1} = \frac{1}{4-a}\begin{pmatrix} 2 & -2-2a & 2+a \\ 2 & 2-3a & 2a-2 \\ -1 & 5 & -3 \end{pmatrix}$	M1 A1 M1 A1 M1	Evaluating determinant $4 - a$ Finding at least four cofactors Six signed cofactors correct Transposing and dividing by det
	When $a = -1$, $\mathbf{M}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 5 & -4 \\ -1 & 5 & -3 \end{pmatrix}$	A1	$\mathbf{M}^{-1}$ correct (in terms of a) and result for $a = -1$ stated *SR*: After 0 scored, SC1 for $\mathbf{M}^{-1}$ when $a = -1$, obtained correctly with some working
		6	
(ii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 5 & -4 \\ -1 & 5 & -3 \end{pmatrix}\begin{pmatrix} -2 \\ b \\ 1 \end{pmatrix}$	M2	Attempting to multiply $(-2 \;\; b \;\; 1)^{\mathrm{T}}$ by given matrix (M0 if wrong order)
		M1	Multiplying out
	$\Rightarrow \quad x = -\frac{3}{5}, y = b - \frac{8}{5}, z = b - \frac{1}{5}$	A2	A1 for one correct
	OR $4x + y = b - 4$ $x - y = 1 - b$ o.e. M1		Eliminating one unknown in 2 ways Or e.g. $3x + z = b - 2$, $5x = -3$ Or e.g. $3y - 4z = -b - 4$, $5y - 5z = -7$
	M1		Solve to obtain one value. Dep. on M1 above
	$\Rightarrow \quad x = -\frac{3}{5}$ A1		One unknown correct After M0, SC1 for value of x
	M1		Finding the other two unknowns
	$\Rightarrow \quad y = b - \frac{8}{5}, z = b - \frac{1}{5}$ A1		Both correct
		5	
(iii)	e.g. $3x - 3y = 2b + 2$ $5x - 5y = 4$	M1 A1A1	Eliminating one unknown in 2 ways Two correct equations Or e.g. $3x + 6z = b - 2$, $5x + 10z = -3$ Or e.g. $3y + 6z = -b - 4$, $5y + 10z = -7$
	Consistent if $\frac{2b+2}{3} = \frac{4}{5}$	M1	Attempting to find b
	$\Rightarrow \quad b = \frac{1}{5}$	A1	
	Solution is a line	B2	
		7	**18**

4 (i)	$\sinh x = \dfrac{e^x - e^{-x}}{2} \Rightarrow \sinh^2 x = \dfrac{\left(e^x - e^{-x}\right)^2}{4}$		
	$= \dfrac{e^{2x} - 2 + e^{-2x}}{4}$	B1	$e^{2x} - 2 + e^{-2x}$
	$\Rightarrow \quad 2\sinh^2 x + 1 = \dfrac{e^{2x} - 2 + e^{-2x}}{2} + 1$		
	$= \dfrac{e^{2x} + e^{-2x}}{2} = \cosh 2x$	B1	Correct completion
	$\Rightarrow \quad 2 \sinh 2x = 4 \sinh x \cosh x$	B1	Both correct derivatives
	$\Rightarrow \quad \sinh 2x = 2 \sinh x \cosh x$	B1	Correct completion
		4	
(ii)	$2\cosh 2x + 3\sinh x = 3$		
	$\Rightarrow \quad 2(1 + 2\sinh^2 x) + 3\sinh x = 3$	M1	Using identity
	$\Rightarrow \quad 4\sinh^2 x + 3\sinh x - 1 = 0$	A1	Correct quadratic
	$\Rightarrow \quad (4\sinh x - 1)(\sinh x + 1) = 0$	M1	Solving quadratic
	$\Rightarrow \quad \sinh x = \frac{1}{4}, -1$	A1	Both
		M1	Use of arsinh $x = \ln(x + \sqrt{x^2+1})$ o.e. Must obtain at least one value of x
	$\Rightarrow \quad x = \text{arsinh}(\frac{1}{4}) = \ln(\dfrac{1+\sqrt{17}}{4})$	A1	Must evaluate $\sqrt{x^2+1}$
	$x = \text{arsinh}(-1) = \ln(-1+\sqrt{2})$	A1	
	OR $2e^{4x} + 3e^{3x} - 6e^{2x} - 3e^x + 2 = 0$		
	$\Rightarrow \quad \left(2e^{2x} - e^x - 2\right)\left(e^{2x} + 2e^x - 1\right) = 0$ M1A1		Factorising quartic
	$\Rightarrow \quad e^x = \dfrac{1 \pm \sqrt{17}}{4}$ or $-1 \pm \sqrt{2}$ M1A1		Solving either quadratic
	$\Rightarrow \quad x = \ln(\dfrac{1+\sqrt{17}}{4})$ or $\ln(-1+\sqrt{2})$ M1A1A1		Using ln (dependent on first M1)
		7	
(iii)	$\cosh t = \dfrac{5}{4} \Rightarrow \dfrac{e^t + e^{-t}}{2} = \dfrac{5}{4}$		
	$\Rightarrow \quad 2e^{2t} - 5e^t + 2 = 0$	M1	Forming quadratic in e^t
	$\Rightarrow \quad \left(2e^t - 1\right)\left(e^t - 2\right) = 0$	M1	Solving quadratic
	$\Rightarrow \quad e^t = \dfrac{1}{2}, 2$	A1	
	$\Rightarrow \quad t = \pm\ln 2$	A1 (ag)	Convincing working
	$\displaystyle\int_4^5 \frac{1}{\sqrt{x^2-16}}\,dx = \left[\text{arcosh}\frac{x}{4}\right]_4^5$	B1	
	$= \text{arcosh}\,\dfrac{5}{4} - \text{arcosh}\,1$	M1	Substituting limits
	$= \ln 2$	A1	A0 for $\pm\ln 2$
	OR $\displaystyle\int_4^5 \frac{1}{\sqrt{x^2-16}}\,dx = \left[\ln\left(x + \sqrt{x^2-16}\right)\right]_4^5$ B1		
	$= \ln 8 - \ln 4$ M1		Substituting limits
	$= \ln 2$ A1		
		7	18

5 (i)	Horz. projection of QP = $k\cos\theta$ Vert. projection of QP = $k\sin\theta$ Subtract OQ = $\tan\theta$	B1 B1 B1 **3**	 Clearly obtained
(ii)	$k = 2$  $k = 1$  $k = ½$  $k = -1$ 	G1 G1 G1 G1 **4**	Loop Cusp
(iii)(A) **(B)** **(C)**	for all k, y axis is an asymptote $k = 1$ $k > 1$	B1 B1 B1 **3**	Both
(iv)	Crosses itself at (1, 0) $k = 2 \Rightarrow \cos\theta = ½ \Rightarrow \theta = 60°$ $\Rightarrow$ curve crosses itself at 120°	 M1 A1 **2**	 Obtaining a value of θ Accept 240°
(v)	$y = 8\sin\theta - \tan\theta$ $\Rightarrow \dfrac{dy}{d\theta} = 8\cos\theta - \sec^2\theta$ $\Rightarrow 8\cos\theta - \dfrac{1}{\cos^2\theta} = 0$ at highest point $\Rightarrow \cos^3\theta = \dfrac{1}{8} \Rightarrow \cos\theta = \pm\dfrac{1}{2} \Rightarrow \theta = 60°$ at top $\Rightarrow x = 4$ $y = 3\sqrt{3}$	 M1 A1 A1 **3**	 Complete method giving θ Both
(vi)	RHS = $\dfrac{(k\cos\theta-1)^2}{k^2\cos^2\theta}\left(k^2-k^2\cos^2\theta\right)$ $= \dfrac{(k\cos\theta-1)^2}{k^2\cos^2\theta}\times k^2\sin^2\theta$ $= (k\cos\theta-1)^2\tan^2\theta$ $= ((k\cos\theta-1)\tan\theta)^2$ $= (k\sin\theta-\tan\theta)^2$ = LHS	M1 M1 E1 **3**	Expressing one side in terms of θ Using trig identities **18**

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General Comments

It is pleasing to report an increase of about a quarter in the number of candidates for this paper, as compared with January 2009. There was no evidence that these extra candidates had been incorrectly entered: again, the number of candidates scoring under 20 marks was well below 10% of the total entry, and the mean mark was almost identical to January 2009, although the standard deviation was a little greater. There was a great deal of very good work, with nearly a quarter of candidates scoring 60 marks or more.

Even competent candidates sometimes succumbed to quite frightening errors in elementary algebra or calculus. These included: confusion of differentiation and integration (the derivative of 1 in Q4(i) often appeared as x); "elementary" differentiation errors (e.g. the derivative of e^t appeared as te^{t-1}); poor use of the laws of logarithms (e.g. $e^t + e^{-t} = 2.5$ in Q4 was followed by $t - t = \ln(2.5)$ or $t + \frac{1}{t} = \ln(2.5)$), ignorance of the laws of algebraic fractions (e.g. in Q2(c) $\frac{t}{t+\frac{1}{2}t^2}$ was very frequently followed by $\frac{t}{t}+\frac{t}{\frac{1}{2}t^2}=1+\frac{2}{t}$ or similar) and other wishful thinking (e.g. in Q1(a) $\int_0^1 \frac{1}{\sqrt{x}\,(x+1)}dx = \int_0^1 \frac{1}{\sqrt{x}}dx \times \int_0^1 \frac{1}{x+1}dx$ or $\frac{1}{\sqrt{x}}\int_0^1 \frac{1}{x+1}dx$). On the other hand, some processes were handled with remarkable efficiency: these included the inverse matrix in Q3(i) and the hyperbolic equation in Q4(ii). Questions 3 and 4 were slightly better done than Questions 1 and 2; Question 5 (Investigations of Curves) was attempted by only one candidate. There was little evidence of time trouble, although as usual some candidates used very inefficient methods to answer some parts of questions: this was particularly evident in Q1(b)(ii), Q2(b) and in some parts of Q4.

Presentation was generally good although once again there were candidates who split questions up and scattered them around the paper, and others who used up to three eight-page answer books. Candidates who used supplementary answer sheets often tagged them in the middle of their main answer book: it is much easier to mark the paper (and sometimes to turn the pages) if these are tagged at the end.

Comments on Individual Questions

1 Calculus of inverse trigonometric functions, polar curves

The mean mark for this question was just under 11.

(a) While there were correct and efficient solutions, very many candidates thought the derivative of $\arctan\sqrt{x}$ was $\frac{1}{1+x}$. This caused problems with the integral, with many candidates blundering ahead in the ways described above. The fact that the answer was given did not deter some candidates from producing $\frac{\pi}{4}$ or ln 2.

(b) The majority of candidates knew how to convert from Cartesian to polar co-ordinates, which was an improvement on previous series. Some of those candidates could not then handle the algebra required to obtain the printed answer, with $\frac{1}{2}$s becoming 2s and the like.

The responses to the second part were very varied. Many candidates did use properties of the sine function, but many of these thought that the minimum value of sin 2θ was 0, and others confused θ and 2θ. Worrying statements like "sin 2θ = 2", sometimes followed by or following "sin θ = 1", were seen fairly frequently. Many gave values of r^2 rather than of r. Other candidates differentiated, either implicitly or (worse) the square root of the given right hand side: this sometimes led to the correct answers, but more frequently to a mess which sometimes covered two pages. Others produced a table: if the correct increment for θ was chosen (by chance?) this sometimes led to an answer which could be credited.

The sketch was usually well done, although some of the ellipses acquired dents or sharp points, and other candidates were determined to draw "well-known" polar curves such as cardioids.

2 Complex numbers

The mean mark for this question was 11.

(a) This part was well answered by the majority of candidates, and it was pleasant to see the efficient methods many of them used. It is quite acceptable to replace cos θ by c and sin θ by s. One common starting point was cos 5θ = $(\cos\theta + \text{j}\sin\theta)^5$: this was condoned, but it is not correct. Some candidates were determined to use the $z + \frac{1}{z}$ method, which was not appropriate. Errors included: minor mistakes with binomial coefficients or signs; ignoring the sine terms completely when equating real parts; including the sine terms as part of the constants a, b and c, and omitting j altogether from the expansion.

(b) Candidates generally did a lot more work than was required in this part. Most recognised what was required at the beginning, writing the series C + jS in exponential form and recognising a geometric series, and many obtained a correct expression for its sum, although finding a sum to infinity was very common and a substantial number experienced trouble in finding the common ratio, which often lost its j. Those who obtained any sort of answer then often spent several pages trying to "realise the denominator": this often included reconversion to trigonometrical form, the use of addition formulae and appeals to the periodicity of sine and cosine. All that was required was to show that the sum was 0, so it was sufficient to show that the numerator was 0, which followed straight from $e^{2\pi\text{j}} = 1$: only a small minority of candidates appreciated this.

(c) Although the required Maclaurin series appears in the formula book (and was often correctly quoted) and the stem of the question was "Write down", many candidates spent time deriving it, sometimes incorrectly. Then most candidates could get as far as substituting the series into the given expression, obtaining $\frac{t}{t+\frac{1}{2}t^2}$ or $\frac{1}{1+\frac{1}{2}t}$. A minority were able to proceed appropriately, either using the binomial theorem or an elegant argument involving multiplying by $1 - \frac{1}{2}t$ or equivalent. Unfortunately the majority, having reached this stage, started blundering about (see above) or just wrote down the given answer with no linking statements. Some candidates just substituted a few numbers, or omitted the part altogether. Quite a few converted the expression to one involving e^{-t}: this required great care in implementation, but was sometimes successful.

3 Matrices

The mean mark for this question was just under 12. (i) was done very well, and (ii) was very often done well, while (iii) proved much more of a challenge.

(i) Finding the inverse of a 3 × 3 matrix caused little problem to the vast majority of candidates: indeed, some were able to almost "write down" the answer, which was most impressive. There were the usual minor sign errors: for some reason the top element of the middle column was most often incorrect. A few candidates multiplied the cofactors by the elements of the original matrix, while one or two set $a = -1$ at the start, and derived the given inverse without involving a: this attracted one mark if done correctly.

(ii) Many candidates used the inverse in (i) and obtained the answer with the minimum of fuss, although some forgot to divide by 5 or made other slips. Those who tried to solve the equations from scratch were often successful as well, although as usual many eliminated x, then y, then z, obtaining three equations in two unknowns from which they could not make progress.

(iii) This part was much less well done. Some tried to use the inverse matrix, setting $a = 4$ and ignoring the inconvenient determinant. Others tried to use the solution to part (ii), ignoring the fact that the system of equations had changed: many of these appeared to believe that, because the second and third equations had not changed, neither would the values of y or z, so if x were eliminated, they could proceed to find b. Others still were determined to use their knowledge of eigenvalues and eigenvectors somewhere, and proceeded to try to find and solve the characteristic equation. Those who did try to eliminate one unknown in two different ways frequently made slips, often when subtracting negative quantities or in working with fractions, so the correct value of b was not seen very often.

The geometrical interpretation was rather badly done. "Line" or "sheaf" were seen (with "sheaf" sometimes becoming "sheath" or even "shaft") but more frequently candidates asserted that the planes intersected at a unique point, or formed a triangular prism, or made self-contradictory statements such as "the planes all intersect along a line and two of them are parallel". Many candidates omitted this part.

4 Hyperbolic functions

This question was, by a small margin, the best answered, with a mean mark of 12.

(i) The proof was done well, although weaker candidates did not always "prove from definitions involving exponentials" and used other (unproven) identities. The second part required candidates to differentiate the identity with respect to x and obtain $2\sinh 2x = 4 \sinh x \cosh x$ in the first instance. This rarely occurred. Many went back to the exponential definitions and differentiated or otherwise manipulated those: those who did attempt to differentiate the identity often lost the 2 on the left, and the right-hand side defeated many completely. Many candidates omitted this part altogether.

(ii) This equation was often solved very efficiently and was a very good source of marks for candidates. Most picked up the clue from (i), although a small minority converted everything to exponentials, obtaining a quartic which could not be factorised, although several almost managed to. Once values of arsinh *x* had been obtained, many candidates went back to first principles, obtaining and solving quadratics in e^x to find *x*; it would have been enough to quote and apply the logarithmic form of arsinh from the formula book.

(iii) This part gave candidates a value of cosh *t* and asked them to show "using exponential functions" that *t* was ±ln 2; it was not acceptable here to quote from the formula book (with or without remarks about cosh *t* being an even function) but several did. The vast majority proceeded in a more appropriate way, although the manner in which –ln 2 was obtained was sometimes rather opaque. A few candidates verified the given result, which was quite acceptable as long as it involved exponential functions. The integral was usually done well although the final answer was given as ±ln 2 rather frequently. One small point is that, although the integral has two forms in the formula book, it is not true that $\text{arcosh}\,\frac{x}{4} = \ln(x + \sqrt{x^2 - 16})$, which was commonly asserted.

5 Investigations of Curves

Only one candidate attempted this question this series.